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On some results concerning the polygonal polynomials

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ABSTRACT.

In this paper we define the nth polygonal polynomial $P_n(z) = (z-1)(z^2-1)\cdots(z^n-1)$ and we investigate recurrence relations and exact integral formulae for the coefficients of P_n and for those of the Mahonian polynomials $Q_n(z) = (z+1)(z^2+z+1)\cdots(z^{n-1}+\cdots+z+1)$. We also explore numerical properties of these coefficients, unraveling new meanings for old sequences and generating novel entries to the Online Encyclopedia of Integer Sequences (OEIS). Some open questions are also formulated.

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