

Kottman's constant, packing constant and Riesz angle in some classes of Köthe sequence spaces

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ABSTRACT.

We have found a sufficient condition in order that the Kottman constant to be equal to the Riesz angle for Köthe sequence spaces. We have found the ball packing constant in weighted Orlicz sequence spaces, endowed with Luxemburg or p -Amemiya norm. We have calculated the Riesz angle for Musielak–Orlicz, Nakano, weighted Orlicz, Orlicz, Orlicz–Lorentz, Lorentz and Cesaro sequence spaces.

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Received: 12.01.2018; In revised form: 30.05.2018; Accepted: 07.06.2018

2010 Mathematics Subject Classification. 46B20, 46E30, 46B45, 46A45.

Key words and phrases. Köthe sequence spaces, Musielak–Orlicz sequence space, weighted Orlicz sequence space, Luxemburg norm, Amemiya norm, Kottman's constant, Riesz angle, Packing constant.

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