

Ulam stability with respect to a directed graph for some fixed point equations

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ABSTRACT.

In this paper, we introduce a new concept of Ulam stability of fixed point equation with respect to a directed graph. Two fixed point theorems of Matkowski and of Jachymski are discussed further in the sense of this stability concept. Some examples about the validity of our notion are given. Finally, we discuss the vagueness of the recent stability results of Sintunavarat [Sintunavarat, W., *A new approach to α - ψ -contractive mappings and generalized Ulam–Hyers stability, well-posedness and limit shadowing results*, Carpathian J. Math., **31** (2015), 395–401].

Acknowledgements. The authors would like to thank the two referees for their comments and suggestions which enhance the presentation of this paper. The corresponding author's research was supported by the Thailand Research Fund and Khon Kaen University under grant RSA5980006.

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Received: 13.07.2018; In revised form: 10.01.2019; Accepted: 15.01.2019

2010 Mathematics Subject Classification. 47H10, 54H25, 39B82.

Key words and phrases. *Ulam–Hyers stability, Ulam stability, fixed point equation, contraction with respect to a directed graph.*

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