

Ulam stability with respect to a directed graph for some fixed point equations

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ABSTRACT.

In this paper, we introduce a new concept of Ulam stability of fixed point equation with respect to a directed graph. Two fixed point theorems of Matkowski and of Jachymski are discussed further in the sense of this stability concept. Some examples about the validity of our notion are given. Finally, we discuss the vagueness of the recent stability results of Sintunavarat [Sintunavarat, W., *A new approach to α - ψ -contractive mappings and generalized Ulam–Hyers stability, well-posedness and limit shadowing results*, Carpathian J. Math., **31** (2015), 395–401].

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