

## Bounds for the skew Laplacian spectral radius of oriented graphs

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### ABSTRACT.

We consider the skew Laplacian matrix of a digraph  $\vec{G}$  obtained by giving an arbitrary direction to the edges of a graph  $G$  having  $n$  vertices and  $m$  edges. We obtain an upper bound for the skew Laplacian spectral radius in terms of the adjacency and the signless Laplacian spectral radius of the underlying graph  $G$ . We also obtain upper bounds for the skew Laplacian spectral radius and skew spectral radius, in terms of various parameters associated with the structure of the digraph  $\vec{G}$  and characterize the extremal graphs.

**Acknowledgements.** The authors thank the anonymous referees for their valuable comments and suggestions which improved the presentation of the paper. The research of S. Pirzada is supported by the SERB-DST research project number EMR/2015/001047 and MTR/2017/000084.

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Received: 28.02.2018; In revised form: 03.12.2018; Accepted: 10.12.2018

2010 Mathematics Subject Classification. 05C50, 05C69, 05C70.

Key words and phrases. Digraph, skew Laplacian matrix, skew Laplacian spectrum.

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