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Bounds for the skew Laplacian spectral radius of oriented graphs

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Abstract.

We consider the skew Laplacian matrix of a digraph $\overrightarrow{G}$ obtained by giving an arbitrary direction to the edges of a graph G having n vertices and m edges. We obtain an upper bound for the skew Laplacian spectral radius in terms of the adjacency and the signless Laplacian spectral radius of the underlying graph G. We also obtain upper bounds for the skew Laplacian spectral radius and skew spectral radius, in terms of various parameters associated with the structure of the digraph $\overrightarrow{G}$ and characterize the extremal graphs.

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