

Dedicated to Professor Romulus Cristescu on the occasion of his 90th birthday

## The Abel-Steffensen inequality in higher dimensions

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### ABSTRACT.

The Abel-Steffensen inequality is extended to the context of several variables. Applications to Fourier analysis and Riemann-Stieltjes integration are included.

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