

# The Abel-Steffensen inequality in higher dimensions

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*Dedicated to Professor Romulus Cristescu on the occasion of his 90th birthday*

## ABSTRACT.

The Abel-Steffensen inequality is extended to the context of several variables. Applications to Fourier analysis and Riemann-Stieltjes integration are included.

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Received: 08.08.2018; In revised form: 03.12.2018; Accepted: 10.12.2018

2010 Mathematics Subject Classification. 26D15, 26B25.

Key words and phrases. *Abel-Steffensen inequality, absolutely continuous function, convex function, 2d-increasing function, Riemann-Stieltjes integral.*