

# A new hybrid algorithm for global minimization of best proximity points in Hilbert spaces

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## ABSTRACT.

The purpose of this paper is to introduce a new hybrid algorithm for finding a global minimization of best proximity points for a new class of mappings, called best proximally nonexpansive (BPNE), which is weaker than nonself nonexpansive mappings and then prove strong convergence of the proposed method under some suitable conditions in real Hilbert spaces. Finally, some numerical experiment is also given for demonstrating our main result.

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