

Dedicated to Prof. Juan Nieto on the occasion of his 60th anniversary

Cyclic permutations and crossing numbers of join products of two symmetric graphs of order six

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ABSTRACT.

The main purpose of this article is broaden known results concerning crossing numbers for join of graphs of order six. We give the crossing number of the join product $G + D_n$, where the graph G consists of one 5-cycle and of one isolated vertex, and D_n consists on n isolated vertices. The proof is done with the help of software that generates all cyclic permutations for a given number k , and creates a new graph COG for calculating the distances between all $(k - 1)!$ vertices of the graph. Finally, by adding some edges to the graph G , we are able to obtain the crossing numbers of the join product with the discrete graph D_n and with the path P_n on n vertices for other two graphs.

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