

*Dedicated to Prof. Juan Nieto on the occasion of his 60<sup>th</sup> anniversary*

## Weighted G-Drazin inverse for operators on Banach spaces

DIJANA MOSIĆ

### ABSTRACT.

We define an extension of weighted G-Drazin inverses of rectangular matrices to operators between two Banach spaces. Some properties of weighted G-Drazin inverses are generalized and some new ones are proved. Using weighted G-Drazin inverses, we introduce and characterize a new weighted pre-order on the set of all bounded linear operators between two Banach spaces. As an application, we present and study the G-Drazin inverse and the G-Drazin partial order for operators on Banach space.

**Acknowledgments.** The author is supported by the Ministry of Education, Science and Technological Development, Republic of Serbia, grant number 174007.

### REFERENCES

- [1] Burgos, M., Márquez-García, A. C. and Morales-Campoy, A., *Maps preserving the diamond partial order*, Appl. Math. Comput., **296** (2017), 137–147
- [2] Castro González, N. and Koliha, J. J., *New additive results for the g-Drazin inverse*, Proc. Roy. Soc. Edinburgh Sect. A, **134** (2004), 1085–1097
- [3] Cline, R. E. and Greville, T. N. E., *A Drazin inverse for rectangular matrices*, Linear Algebra Appl., **29** (1980), 53–62
- [4] Coll, C., Lattanzi, M. and Thome, N., *Weighted G-Drazin inverses and a new pre-order on rectangular matrices*, Appl. Math. Comput., **317** (2018), 12–24
- [5] Dajić, A. and Koliha, J. J., *The weighted g-Drazin inverse for operators*, J. Australian Math. Soc., **82** (2007), 163–181
- [6] Hernández, A., Lattanzi, M. and Thome, N., *On some new pre-orders defined by weighted Drazin inverses*, Appl. Math. Comput., **282** (2016), 108–116
- [7] Hernández, A., Lattanzi, M. and Thome, N., *Weighted binary relations involving the Drazin inverse*, Appl. Math. Comput., **253** (2015), 215–223
- [8] Koliha, J. J., *A generalized Drazin inverse*, Glasgow Math. J., **38** (1996), 367–381
- [9] Marovt, J., *On star, sharp, core and minus partial orders in Rickart rings*, Banach J. Math. Anal., **10** (2016), No. 3, 495–508
- [10] Mitra, S. K., Bhimasankaram, P. and Malik, S. B., *Matrix partial orders, shorted operators and applications*, World Scientific Publishing Company, 2010
- [11] Mosić, D., *Extensions of Jacobson's lemma for Drazin inverses*, Aequat. Math., **91** (2017), No. 3, 419–428
- [12] Mosić, D., *Reverse order laws on the conditions of the commutativity up to a factor*, Revista de La Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas, **111** (2017), 685–695
- [13] Mosić, D., *Weighted binary relations for operators on Banach spaces*, Aequat. Math., **90** (2016), No. 4, 787–798
- [14] Mosić, D. and Djordjević, D. S., *Some additive results for the Wg-Drazin inverse of Banach space operators*, Carpathian J. Math., **32** (2016), No. 2, 215–223
- [15] Mosić, D. and Djordjević, D. S., *Weighted pre-orders involving the generalized Drazin inverse*, Appl. Math. Comput., **270** (2015), 496–504
- [16] Rakočević, V. and Wei, Y., *A weighted Drazin inverse and applications*, Linear Algebra Appl., **350** (2002), 25–39

Received: 01.08.2018; In revised form: 10.01.2019; Accepted: 17.01.2019

2010 Mathematics Subject Classification. 47A05, 47A62, 47A99, 15A09.

Key words and phrases. *weighted G-Drazin inverse, Wg-Drazin inverse, G-Drazin partial order, minus partial order.*

- [17] Robles, J., Martinez-Serrano, M. F. and Dopazo, E., *On the generalized Drazin inverse in Banach algebras in terms of the generalized Schur complement*, Appl. Math. Comput., **284** (2016), 162–168
- [18] Srivastava, S., Gupta, D. K., Stanimirović, P. S., Singh, S. and Roy, F., *A hyperpower iterative method for computing the generalized Drazin inverse of Banach algebra element*, Sādhanā, **42** (2017), No. 5, 625–630
- [19] Wang, H. and Liu, v., *Partial orders based on core-nilpotent decomposition*, Linear Algebra Appl., **488** (2016), 235–248
- [20] Wang, X. Z., Ma, H. and Stanimirović, P. S., *Recurrent neural network for computing the W-weighted Drazin inverse*, Appl. Math. Comput., **300** (2017), 1–20
- [21] Zhang, X. and Sheng, X., *Two methods for computing the Drazin inverse through elementary row operations*, Filomat, **30** (2016), No. 14, 3759–3770

UNIVERSITY OF NIŠ  
FACULTY OF SCIENCES AND MATHEMATICS  
P.O. BOX 224, 18000 NIŠ, SERBIA  
E-mail address: dijana@pmf.ni.ac.rs