Dedicated to Prof. Juan Nieto on the occasion of his 60th anniversary

Coupled fixed point theorems in quasimetric spaces without mixed monotonicity

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ABSTRACT.

In this paper, using the concepts of f-closed set and inverse f-closed set, we will prove some fixed point theorems for graphic contractions in complete quasimetric space. Then, as applications, coupled fixed point theorems in quasimetric spaces without the mixed monotonicity property are obtained.

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