Dedicated to Prof. Juan Nieto on the occasion of his 60th anniversary

Existence of tripled fixed points and solution of functional integral equations through a measure of noncompactness

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ABSTRACT.

In this paper, we propose fixed point results through the notion of a measure of noncompactness and give a generalization of a Darbo's fixed point theorem. We also prove some new tripled fixed point results via a measure of noncompactness for a more general class of functions. Our results generalize and extend some comparable results in the literature. Further, we apply the obtained fixed point theorems to prove the existence of solutions for a general system of non-linear functional integral equations. In the end, an example is given to illustrate the validity of our results.

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