Dedicated to Prof. Juan Nieto on the occasion of his 60th anniversary

Convergence results for fixed point iterative algorithms in metric spaces

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Abstract.
Let \((X, d)\) be a metric space, \(f, f_n : X \to X\), with \(F_f = F_{f_n}, n \in \mathbb{N}\). For the fixed point equation
\[ x = f(x) \]
we consider the following iterative algorithm,
\[ x \in X, \ x_0 = x, \ x_{n+1}(x) = f_n(x_n(x)), \ n \in \mathbb{N}. \]
By definition, the algorithm \(2\) is convergent if,
\[ x_n(x) \to x^*(x) \in F_f \text{ as } n \to \infty, \forall \ x \in X. \]
In this paper we give some conditions on \(f_n\) and \(f\) which imply the convergence of algorithm \(2\). In this way we improve some results given in [Rus, I. A., An abstract point of view on iterative approximation of fixed points: impact on the theory of fixed point equations, Fixed Point Theory, 13 (2012), No. 1, 179–192]. In our results, in general we do not suppose that, \(F_f \neq \emptyset\). Some research directions are formulated.

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