

Dedicated to Prof. Juan Nieto on the occasion of his 60th anniversary

Some new results of M-iteration process in hyperbolic spaces

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ABSTRACT.

In this paper, we study the M-iteration process in hyperbolic spaces and prove some strong and Δ -convergence theorems of this iteration process for generalized nonexpansive mappings. Moreover, we establish the weak w^2 -stability and data dependence theorems for a class of contractive-type mappings by using M-iteration process. The results presented here extend and improve some recent results announced in the current literature.

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