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Dedicated to Prof. Juan Nieto on the occasion of his 60^{th} anniversary

Behaviour of advection-diffusion-reaction processes with forcing terms

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ABSTRACT.
Without doing any linearization, this paper mainly focuses on capturing numerical behavior of the advection-diffusion-reaction (ADR) processes with forcing terms. Since the linearization of nonlinear systems loses real features, the physical systems are important to understand their natural properties. Therefore we concentrate on investigation of the real-world processes without losing their properties. To achieve the aforementioned aims, this article presents two newly combined methods; the backward differentiation formula-Spline (BDFS) and the optimal five stage and fourth-order strong stability preserving Runge-Kutta-Spline (SSPRK54S) methods. In the current methods, neither linearization nor transforming the process is required. Comparison between the two methods is carried out in dealing with the ADR problems to check the efficiency and utility of the proposed schemes. Accuracy of the methods is assessed in terms of the relative and absolute errors. The computed results showed that the BDFS method is seen to be more powerful, quite accurate and more economical in comparison with the SSPRK54S method. The current method is seen to be a very reliable alternative in solving the problem by conserving the physical properties of the nature. The BDFS method is realized to be efficient for these types of physical problems and be easy to implement. The results have revealed that the BDFS scheme is relatively free of choice of the physical parameters.

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