

Dedicated to Prof. Juan Nieto on the occasion of his 60th anniversary

Sequential characterizations of robust optimal solutions in uncertain convex programs via perturbation approach

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ABSTRACT.

By using robust optimization approach, necessary and sufficient sequential optimality conditions without any constraint qualifications for the general convex optimization problem in the face of data uncertainty are given in terms of the ε -subdifferential. A sequential condition involving only the subdifferentials is also derived using a version of the Brøndsted-Rockafellar theorem. Consequently, sequential Lagrange multiplier condition for robust optimal solution of convex optimization problem with cone constraints in the face of data uncertainty is given. It is worth pointing out that there is no compactness assumption of uncertainty set and upper semicontinuity of functions involved in our results.

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