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Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60^{th} anniversary

The Opial condition in variable exponent sequence spaces $\ell_{p(\cdot)}$ with applications

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ABSTRACT.

In this work, we show an analogue to the Opial property for the coordinate-wise convergence in the variable exponent sequence space $\ell_{p(\cdot)}$. This property allows us to prove a fixed point theorem for the mappings which are nonexpansive in the modular sense.

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REFERENCES

[1] Browder, F. E. and Petryshyn, W. V., *The solution by iteration of nonlinear functional equations in Banach spaces*, Bull. Amer. Math. Soc., **72** (1966), 571–575
[2] Domnguez-Benavides, T., Garca-Falset, J. and Japón Pineda, M., *The τ-fixed point property for nonexpansive mappings*, Abstr. Appl. Anal., **3** (1998), 343–362
[3] Diening, L., Harjulehto, P., Hästö, P. and Ružîčka, M., *Lebesgue and Sobolev Spaces with Variable Exponents*, Lecture Note in Mathematics, Springer, Berlin, (2017)
[4] Khamsi, M. A and Kirk, W. A., *An Introduction to Metric Spaces and Fixed Point Theory*, John Wiley, New York, (2001)
[5] Khamsi, M. A. Kozlowski, W. K. and Reich, S., *Fixed point theory in modular function spaces*, Nonlinear Analysis, **14** (1990), 935–953
[6] Khamsi, M. A. and Kozlowski, W. M., *Fixed Point Theory in Modular Function Spaces*, Birkhauser New York, (2015)
[7] Klee, V., *Summability in $\ell(p_{11}, p_{21}, \cdots)$ Spaces*, Studia Math., (1965), 277–280
[8] Kováčik, O. and Rákosník, J., *On spaces $L^{p(x)}$ and $W^{1,p(x)}$*, Czechoslovak Math. J., **41** (1991), No. 116, 592–618
[9] Kozlowski, W. M., *Modular Function Spaces*, Series of Monographs and Textbooks in Pure and Applied Mathematics, **122**, Dekker, New York/Basel, (1988)
[10] Lim, T. C., *Asymptotic centers and nonexpansive mappings in conjugate Banach spaces*, Pacific J. Math., **90** (1980), 135–143
[11] Musielak, J., *Orlicz spaces and modular spaces*, Lecture Notes in Mathematics, Vol.**1034**, Springer-Verlag, Berlin/Heidelberg/New York/Tokyo, (1983)
[12] Nakano, H., *Modulared Semi-ordered Linear Spaces*, Maruzen Co., Tokyo, (1950)
[13] Nakano, H., *Topology of linear topological spaces*, Maruzen Co. Ltd., Tokyo, (1951)
[14] Nakano,H., *Modulared sequence spaces*, Proc. Japan Acad., **27** (1951), 508–512

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[15] Opial, Z., *Weak convergence of the sequence of successive approximations for nonexpansive mappings*, Bull. Amer. Math. Soc., **73** (1931), 591-597

[16] Orlicz, W., *Über konjugierte Exponentenfolgen*, Studia Math., (1931), 200–211

[17] Rajagopal, K. and Ružička, M., *On the modeling of electrorheological materials*, Mech. Research Comm., **23** (1996), 401–407

[18] Ružička, M., *Electrorheological fluids: modeling and mathematical theory*, Lecture Notes in Mathematics 1748, Springer-Verlag, Berlin (2000)

[19] Sundaresan, K., *Uniform convexity of Banach spaces* $\ell(\{p_i\})$, Studia Mathematica., **39** (1971), 227–231

[20] Waterman, D., Ito, T., Barber, F. and Ratti, J., *Reflexivity and Summability: The Nakano* $\ell(p_i)$ *spaces*, Studia Math., **331** (1969), 141–146

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