

Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60th anniversary

Intersection theorems with applications in set-valued equilibrium problems and minimax theory

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ABSTRACT.

In this paper, we obtain three intersection theorems that can be considered versions of Theorem 3.1 from the paper [Agarwal, R. P., Balaj, M. and O'Regan, D., *Intersection theorems with applications in optimization*, J. Optim. Theory Appl., **179** (2018), 761–777]. As will be seen, there are two major differences between the hypotheses of the above mentioned theorem and those of our results. Applications of the main results are considered in the last two sections of the paper.

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