

Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60th anniversary

Approximation of solutions of Hammerstein equations with monotone mappings in real Banach spaces

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ABSTRACT.

Let E be a uniformly convex and uniformly smooth real Banach space with dual space, E^* . Let $F : E \rightarrow E^*$, $K : E^* \rightarrow E$ be maximal monotone mappings. An iterative algorithm is constructed and the sequence of the algorithm is proved to converge strongly to a solution of the Hammerstein equation $u + KF u = 0$. This theorem is a significant improvement of some important recent results which were proved in L_p spaces, $1 < p \leq 2$ under the assumption that F and K are bounded. This restriction on K and F have been dispensed with even in the more general setting considered here. Finally, a numerical experiment is presented to illustrate the convergence of the sequence of the algorithm which is found to be much faster, in terms of the number of iterations and the computational time than the convergence obtained with existing algorithms.

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