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Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60^{th} anniversary

Strong convergence of inertial subgradient extragradient method for solving variational inequality in Banach space

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ABSTRACT.

In this paper, we introduce a modified inertial subgradient extragradient algorithm in a 2-uniformly convex and uniformly smooth real Banach space and prove a strong convergence theorem for approximating a common solution of fixed point equation with a demigeneralized mapping and a variational inequality problem of a monotone and Lipschitz mapping. We present an example to validate our new findings. This work substantially improves and generalizes some well-known results in the literature.

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