

Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60<sup>th</sup> anniversary

## A nonsmooth Stackelberg equilibrium problem via mixed variational inequalities

JUE LU, LI-WEN ZHOU, YI-BIN XIAO and NAN-JING HUANG

### ABSTRACT.

In this paper, we introduce and study a class of nonsmooth Stackelberg equilibrium problems with differential revenue functions and nonsmooth cost functions. First, we describe the best response set of the follower and the best strategy set of the leader for the considered nonsmooth Stackelberg equilibrium problem via mixed variational inequalities. Then, by using the resolvent operator method and the Banach fixed point theorem, we show the existence and uniqueness of its solution.

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Corresponding author: Nan-jing Huang; nanjinghuang@hotmail.com

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SHAOXING UNIVERSITY  
SCHOOL OF MATHEMATICS  
PHYSICS AND INFORMATION SCIENCE  
HUANCHENG WEST RD. NO.508, 312000, SHAOXING, P. R. CHINA  
E-mail address: admiral.lu@hotmail.com

SOUTHWEST PETROLEUM UNIVERSITY  
SCHOOL OF SCIENCES, XINDU ROAD 8, 610500, CHENGDU, P. R. CHINA  
E-mail address: zhouluwen@live.cn

UNIVERSITY OF ELECTRONIC SCIENCE AND TECHNOLOGY OF CHINA  
SCHOOL OF MATHEMATICAL SCIENCES  
WEST HI-TECH.ZONE XIYUAN AVE NO.2006, 611731, CHENGDU, P. R. CHINA  
E-mail address: xiaoyb9999@hotmail.com

DEPARTMENT OF MATHEMATICS  
SICHUAN UNIVERSITY  
WANGJIANG ROAD 29, 610064, CHENGDU, P. R. CHINA  
E-mail address: nanjinghuang@hotmail.com