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Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60^{th} anniversary

A new forward-backward penalty scheme and its convergence for solving monotone inclusion problems

NATTHAPHON ARTSAWANG[1] and KASAMSUK UNGCHITTRAKOOL[1,2]

ABSTRACT.

The purposes of this paper are to establish an alternative forward-backward method with penalization terms called new forward-backward penalty method (**NFBP**) and to investigate the convergence behavior of the new method via numerical experiment. It was proved that the proposed method (**NFBP**) converges in norm to a zero point of the monotone inclusion problem involving the sum of a maximally monotone operator and the normal cone of the set of zeros of another maximally monotone operator. Under the observation of some appropriate choices for the available properties of the considered functions and scalars, we can generate a suitable method that weakly ergodic converges to a solution of the monotone inclusion problem. Further, we also provide a numerical example to compare the new forward-backward penalty method with the algorithm introduced by Attouch [Attouch, H., Czarnecki, M.-O. and Peypouquet, J., *Coupling forward-backward with penalty schemes and parallel splitting for constrained variational inequalities*, SIAM J. Optim., **21** (2011), 1251-1274].

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Corresponding author: Kasamsuk Ungchittrakool; kasamsuku@nu.ac.th

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[1]Department of Mathematics
Faculty of Science
Naresuan University
Phitsanulok 65000, Thailand
E-mail address: na_tta_pon@hotmail.com
E-mail address: kasamsuku@nu.ac.th

[2]RESEARCH CENTER FOR ACADEMIC EXCELLENCE IN NONLINEAR ANALYSIS AND OPTIMIZATION
FACULTY OF SCIENCE, PHITSANULOK 65000, THAILAND
E-mail address: kasamsuku@nu.ac.th