

Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60<sup>th</sup> anniversary

## Limiting proper minimal points of nonconvex sets in finite-dimensional spaces

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### ABSTRACT.

In this paper, limiting proper minimal points of nonconvex sets in Euclidean finite-dimensional spaces are investigated. The relationships between these minimal points and Borwein, Benson, and Henig proper minimal points, under appropriate assumptions, are established. Furthermore, a density property is derived and a linear characterization of limiting proper minimal points is provided.

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