

Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60th anniversary

External and internal stability in set optimization using gamma convergence

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ABSTRACT.

The main objective of this paper is to investigate the stability of solution sets of perturbed set optimization problems in the decision space as well as in the image space, by perturbing the objective maps. For a sequence of set-valued maps, a notion of gamma convergence is introduced to establish the external and internal stability in terms of Painlevé-Kuratowski convergence of sequence of solution sets of perturbed problems under certain compactness assumptions and domination properties.

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