

Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60th anniversary

Characterizing the Lagrange multiplier rule in nonconvex set-valued optimization

S. ATARZADEH¹, M. FAKHAR¹ and J. ZAFARANI²

ABSTRACT.

In this article, by using the notions of contingent derivative, contingent epiderivative and generalized contingent epiderivative, we obtain some characterizations of the Lagrange multiplier rule at points which are not necessarily local minima.

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Corresponding author: M. Fakhari; fakhari@math.ui.ac.ir

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¹DEPARTMENT OF MATHEMATICS

UNIVERSITY OF ISFAHAN

ISFAHAN, 81745-163, IRAN

E-mail address: atarzadeh@shbu.ac.ir

E-mail address: fakhar@sci.ui.ac.ir

²SHEIKHBAHAEI UNIVERSITY AND UNIVERSITY OF ISFAHAN

DEPARTMENT OF MATHEMATICS

ISFAHAN, IRAN

E-mail address: jzaf@zafarani.ir