Dedicated to Prof. Qamrul Hasan Ansari on the occasion of his 60th anniversary

On quasi approximate solutions for nonsmooth robust semi-infinite optimization problems

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ABSTRACT.

This paper devotes to the quasi ε-solution for robust semi-infinite optimization problems (RSIP) involving a locally Lipschitz objective function and infinitely many locally Lipschitz constraint functions with data uncertainty. Under the fulfillment of robust type Guignard constraint qualification and robust type Kuhn-Tucker constraint qualification, a necessary condition for a quasi ε-solution to problem (RSIP). After introducing the generalized convexity, we give a sufficient optimality for such a quasi ε-solution to problem (RSIP). Finally, we also establish approximate duality theorems in term of Wolfe type which is formulated in approximate form.

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