

Dedicated to Prof. Hong-Kun Xu on the occasion of his 60<sup>th</sup> anniversary

## An iterative method for solving multiple-set split feasibility problems in Banach spaces

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### ABSTRACT.

In this paper, we study generalized multiple-set split feasibility problems (in short, GMSSFP) in the frame work of  $p$ -uniformly convex real Banach spaces which are also uniformly smooth. We construct an iterative algorithm which is free from an operator norm and prove its strong convergence to a solution of GMSSFP, that is, a solution of convex problem and a common fixed point of a countable family of Bregman asymptotically quasi-nonexpansive mappings without requirement for semi-compactness on the mappings. We illustrate our algorithm and convergence result by a numerical example.

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