CARPATHIAN J. MATH.
Volume **36** (2020), No. 1,
Pages 109 - 117

Online version at https://www.carpathian.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 2851; Online Edition: ISSN 1843 - 4401
DOI: https://doi.org/10.37193/CJM.2020.01.10

Dedicated to Prof. Hong-Kun Xu on the occasion of his 60^{th} anniversary

Weak sharpness for solutions of nonsmooth variational inequalities and applications

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ABSTRACT.

In this paper, we first give some new characterizations of weak sharpness of the solution set of nonsmooth variational inequalities in terms of partial subdiferentials/Gâteaux derivatives of involving bifunctions. As applications, we use a new characterization to establish sufficient conditions for guaranteeing finite termination of an arbitrary algorithm solving nonsmooth variational inequalities under the weak sharpness assumption.

Acknowledgements. This article was supported by the National Natural Science Foundation of China under Grant No.11401152. The first author was also supported by the Research Fund for International Young Scientists under Grant No. 1181101157.

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Received: 07.05.2019; In revised form: 31.01.2020; Accepted: 07.02.2020
2010 *Mathematics Subject Classification*. 49J40, 90C26, 90C52, 47J20.
Key words and phrases. *Weak sharpness and finite convergence and nonsmooth variational inequalities.*
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