

Dedicated to Prof. Hong-Kun Xu on the occasion of his 60th anniversary

Modified inertial double Mann type iterative algorithm for a bivariate weakly nonexpansive operator

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ABSTRACT.

We introduce a modified inertial double Mann type iterative method to approximate coupled solutions of a bivariate nonexpansive operator $T : C \times C \rightarrow C$, where C is a nonempty closed and convex subset of a Hilbert space. The one theorem and complement important old and recent results in coupled fixed point theory. Some appropriate examples to illustrate our results and their generalization are also given.

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