

Dedicated to Prof. Hong-Kun Xu on the occasion of his 60th anniversary

Fréchet vector subdifferential calculus

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ABSTRACT.

In this paper, we study Fréchet vector subdifferentials of vector-valued functions in normed spaces which reduce to the known ones of extended-real-valued functions. We establish relations between two kinds of Fréchet vector subdifferentials and between subdifferential and coderivative; some of them improve the existing relations for extended-real-valued functions. Finally, sum and chain rules among others for Fréchet subdifferentials of vector-valued functions are formulated and verified. Many examples are provided.

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