

Dedicated to Prof. Hong-Kun Xu on the occasion of his 60th anniversary

Approximating fixed points of enriched nonexpansive mappings in Banach spaces by using a retraction-displacement condition

VASILE BERINDE

ABSTRACT.

In this paper, we prove convergence theorems for a fixed point iterative algorithm of Krasnoselskij-Mann type associated to the class of enriched nonexpansive mappings in Banach spaces. The results are direct generalizations of the corresponding ones in [Berinde, V., *Approximating fixed points of enriched nonexpansive mappings by Krasnoselskij iteration in Hilbert spaces*, Carpathian J. Math., **35** (2019), No. 3, 293–304.], from the setting of Hilbert spaces to Banach spaces, and also of some results in [Senter, H. F. and Dotson, Jr., W. G., *Approximating fixed points of nonexpansive mappings*, Proc. Amer. Math. Soc., **44** (1974), No. 2, 375–380.], [Browder, F. E., Petryshyn, W. V., *Construction of fixed points of nonlinear mappings in Hilbert space*, J. Math. Anal. Appl., **20** (1967), 197–228.], by considering enriched nonexpansive mappings instead of nonexpansive mappings. Many other related results in literature can be obtained as particular instances of our results.

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DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
 TECHNICAL UNIVERSITY OF CLUJ-NAPOCA
 NORTH UNIVERSITY CENTRE AT BAI A MARE
 VICTORIEI 76, RO-430122 BAI A MARE, ROMANIA
 Email address: vberinde@cunbm.utcluj.ro