

Dedicated to Prof. Hong-Kun Xu on the occasion of his 60th anniversary

A class of parabolic evolutionary quasivariational inequalities in contact mechanics

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ABSTRACT.

In this paper, we obtain an existence and uniqueness of the solution for a class of parabolic evolutionary quasivariational inequalities in contact mechanics under some mild conditions. We also study an error estimate for the parabolic evolutionary quasivariational inequality by employing the forward Euler difference scheme and the element-free Galerkin spatial approximation.

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