CARPATHIAN J. MATH.
Volume **36** (2020), No. 1,
Pages 59 - 69

Online version at https://www.carpathian.cunbm.utcluj.ro/
Print Edition: ISSN 1584 - 2851; Online Edition: ISSN 1843 - 4401
DOI: https://doi.org/10.37193/CJM.2020.01.06

Dedicated to Prof. Hong-Kun Xu on the occasion of his 60^{th} anniversary

Approximation of zeros of m-accretive mappings, with applications to Hammerstein integral equations

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ABSTRACT.

An algorithm for approximating zeros of m-accretive operators is constructed in a uniformly smooth real Banach space. The sequence generated by the algorithm is proved to converge strongly to a zero of an m-accretive operator. In the case of a real Hilbert space, our theorem complements the celebrated proximal point algorithm of Martinet and Rockafellar for approximating zeros of maximal monotone operators. Furthermore, the convergence theorem proved is applied to approximate a solution of a Hammerstein integral equation. Finally, numerical experiments are presented to illustrate the convergence of our algorithm.

Acknowledgements. The authors appreciate the support of their institute and the African Development Bank (AfDB) for the Research Grant that enable this work to be carried out. The authors wish to thank the referees for their esteemed comments and suggestions.

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Received: 21.08.2019; In revised form: 27.01.2020; Accepted: 04.02.2020
2010 *Mathematics Subject Classification.* 47H09, 47H10, 47J25 47J05, 47J20.
Key words and phrases. *Fixed points, pseudocontractive mapping, accretive mapping, strong convergence.*
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