Dedicated to Prof. Hong-Kun Xu on the occasion of his 60th anniversary

Existence and approximation of a fixed point of a fundamentally nonexpansive mapping in hyperbolic spaces

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ABSTRACT.

We prove that a fundamentally nonexpansive mapping on a compact and convex subset of a hyperbolic space, has a fixed point. We also show that one–step iterative algorithm of two mappings is vital for the approximation of a common fixed point of two fundamentally nonexpansive mappings in a strictly convex hyperbolic space. Our results are new in metric fixed point theory and generalize several existing results.

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