

Dedicated to Prof. Billy E. Rhoades on the occasion of his 90th anniversary

Fixed point theorems and convergence theorems for some monotone generalized nonexpansive mappings

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ABSTRACT.

We present some new coincidence fixed point theorems for generalized multi-valued weak Γ -contraction mappings. Our outcomes extend several recent results in the framework of complete metric spaces endowed with a graph. Two illustrative examples are included and some consequences are derived.

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