

Dedicated to Prof. Billy E. Rhoades on the occasion of his 90th anniversary

A probabilistic Meir-Keeler type fixed point theorem which characterizes metric completeness

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ABSTRACT.

A probabilistic version of the Meir-Keeler type fixed point theorem, which characterizes completeness of the metric space is established. In addition to it, a fixed point theorem for non-expansive mappings satisfying $(\epsilon - \delta)$ type condition in Menger probabilistic metric space (Menger PM-space) is proved. As a byproduct we find an affirmative answer to the open question on the existence of contractive mappings which admit discontinuity at the fixed point (see Rhoades, B. E., *Contractive definitions and continuity*, Contemporary Mathematics 72 (1988), 233–245, p. 242) in the setting of Menger PM-space.

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REFERENCES

- [1] Abtahi, M., *Suzuki-type fixed point theorems for generalized contractive mappings that characterize metric completeness*, Bull. Iranian Math. Soc., **41** (2015), No. 4, 931–943
- [2] Berinde, V. and Choban, M., *Remarks on some completeness conditions involved in several common fixed point theorems*, Creat. Math. Inform., **19** (2010), No. 1, 1–10
- [3] Bisht, R. K. and Pant, R. P., *A remark on discontinuity at fixed point*, J. Math. Anal. Appl., **445** (2017), 1239–1242
- [4] Bisht, R. K. and Rakočević, V., *Generalized Meir-Keeler type contractions and discontinuity at fixed point*, Fixed Point Theory, **19** (2018), No. 1, 57–64
- [5] Ćirić, Lj. B., *On contraction type mappings*, Math. Balkanica, **1** (1971), 52–57
- [6] Hicks, T. and Rhoades, B. E., *Fixed points and continuity for multivalued mappings*, International J. Math. Math. Sci., **15** (1992), 15–30
- [7] Hicks, T. L. and Rhoades, B. E., *A Banach type fixed-point theorem*, Math. Japon., **24** (1979/80), 327–330
- [8] Kadelburg, Z., Radenović, S. and Shukla, S., *Boyd-Wong and Meir-Keeler type theorems in generalized metric spaces*, J. Adv. Math. Stud., **9** (2016), No. 1, 83–93
- [9] Meir, A. and Keeler, E., *A theorem on contraction mappings*, J. Math. Anal. Appl., **28** (1969), 326–329
- [10] Menger, K., *Statistical metric*, Proc. Nat. Acad. Sci. USA, **28** (1942), 535–537
- [11] Mitrović, Z. and Radenović, S., *On Meir-Keeler contraction in Branciari b-metric spaces*, Transactions of A. Razmadze Mathematical Institute, **173** (2019), 83–90
- [12] Nguyen, L. V., *On fixed points of asymptotically regular mappings*, Rend. Circ. Mat. Palermo, II. Ser (to appear)
- [13] Pant, A. and Pant, R. P., *Fixed points and continuity of contractive maps*, Filomat **31** (2017), No. 11, 3501–3506
- [14] Pant, A., Pant, R. P. and Joshi, M. C., *Caristi type and Meir-Keeler type fixed point theorems*, Filomat, **33** (2019), No. 12, 3711–3721
- [15] Pant, R. P., *Discontinuity and fixed points*, J. Math. Anal. Appl., **240** (1999), 284–289
- [16] Pant, R. P., Özgür, N. Y. and Taş, N., *On discontinuity problem at fixed point*, Bull. Malays. Math. Sci. Soc., **43** (2020), No. 1, 499–517
- [17] Pant, R. P., Pant, A., Nikolić, R. M. and Ješić, S. N., *A characterization of completeness of Menger PM-spaces*, J. Fixed Point Theory Appl., **21**, (2019), 90 pp.

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- [18] Pant, R. P., Özgür N. Y. and Taş, N., *Discontinuity at fixed points with applications*, Bulletin of the Belgian Mathematical Society-Simon Stevin, **25** (4), (2019), 571–589
- [19] Popescu, O., *A new type of contractions that characterize metric completeness*, Carpathian J. Math., **31** (2015), No. 3, 381–387
- [20] Rhoades, B. E., *Contractive definitions and continuity*, Contemporary Mathematics, **72** (1988), 233–245
- [21] Schweizer, B. and Sklar, A., *Statistical metric spaces*, Pacific J. Math., **10** (1960), 415–417
- [22] Schweizer, B. and Sklar, A., *Probabilistic Metric Spaces*, North-Holland, New York, Elsevier 1983
- [23] Sehgal, V. M. and Bharucha-Reid, A. T., *Fixed points of contraction mappings in PM-spaces*, Math. System Theory, **6** (1972), 97–102
- [24] Subrahmanyam, P. V., *Completeness and fixed points*, Monatsh. Math., **80** (1975), 325–330
- [25] Suzuki, T., *A generalized Banach contraction principle that characterizes metric completeness*, Proc. Amer. Math. Soc., **136** (2008), No. 5, 1861–1869
- [26] Taş, N. and Özgür, N. Y., *A new contribution to discontinuity at fixed point*, Fixed Point Theory, **20** (2019), No. 2, 715–728
- [27] Todorčević, V., *Harmonic Quasiconformal Mappings and Hyperbolic Type Metric*, Springer Nature Switzerland AG 2019

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