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Dedicated to Prof. Billy E. Rhoades on the occasion of his 90^{th} anniversary

On Caristi's fixed point theorem in metric spaces with a graph

NANTAPORN CHUENSUPANTHARAT and DHANANJAY GOPAL

ABSTRACT.

We generalize the Caristi's fixed point theorem for single valued as well as multivalued mappings defined on a metric space endowed with a graph and w-distance. Particularly, we modify the concept of the (OSC)-property due to Alfuraidan and Khamsi (Alfuraidan M. R. and Khamsi, M. A., *Caristi fixed point theorem in metric spaces with graph*, Abstr. Appl. Anal., (2014) Art. ID 303484, 5.) which enable us to reformulated their stated graph theory version theorem (Theorem 3.2 in Alfuraidan M. R. and Khamsi, M. A., *Caristi fixed point theorem in metric spaces with graph*, Abstr. Appl. Anal., (2014) Art. ID 303484, 5.) to the case of w-distance. Consequently, we extend and improve some recent works concerning extension of Banach Contraction Theorem to w-distance with graph e.g. (Jachymski, J., *The contraction principle for mappings on a metric space with graph*, Proc. Amer. Math. Soc., **136** (2008), No. 4, 1359–1373; Nieto, J. J., Pouso, R. L. and Rodriguez-Lopez R., *Fixed point theorems in ordered abstract spaces*, Proc. Amer. Math. Soc., **135** (2007), 2505–2517 and Petrusel, A. and Rus, I., *Fixed point theorems in ordered $L-$spaces endowed with graph*, Proc. Amer, Math. Soc., **134** (2006), 411–418).

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Corresponding author: Nantaporn Chuensupantharat; nantaporn.chuen@gmail.com

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Department of Mathematics and Statistics
Faculty of Science and Technology
Bansomdejchaopraya Rajabhat University, 1061 Isaraphab 15 Rd., 10600, Thailand
Email address: nantaporn.chuen@gmail.com, gopaldhananjay@yahoo.in; dg@amhd.svnit.ac.in