Dedicated to Prof. Billy E. Rhoades on the occasion of his 90th anniversary

Strong convergence of Picard and Mann iterations for strongly demicontractive multi-valued mappings

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Abstract.

A class of demicontractive mappings was first introduced in [Hicks, T. L. and Kubicek, J. D., *On the Mann iteration process in a Hilbert space*, J. Math. Anal. Appl., **59** (1977) 498–504 and Mǎruşter, Ş., *The solution by iteration of nonlinear equations in Hilbert spaces*, Proc. Amer. Math. Soc., **63** (1977), 69–73] and was first mentioned in the case of multi-valued mappings in [Chidume, C. E., Bello, A. U. and Ndambomve, P., *Strong and Δ-convergence theorems for common fixed points of a finite family of multivalued demicontractive mappings in CAT(0) spaces*, Abstr. Appl. Anal., **2014** (2014), https://doi.org/10.1155/2014/805168 and Isiogugu, F. O. and Osilike, M. O., *Convergence theorems for new classes of multivalued hemicontractive-type mappings*, Fixed Point Theory Appl., **2014** (2014), https://doi.org/10.1186/1687-1812-2014-93]. The demicontractivity with some weak smoothness conditions ensures only weak convergence of Mann iteration. In 2015, Mǎruşter and Rus [*Kannan contractions and strongly demicontractive mappings*, Creat. Math. Inform., **24** (2015), No. 2, 173–182], introduced a class of strongly demicontractive mappings, and also discussed some relationships between strongly demicontractive mappings and Kannan contractions. In this paper, we introduce a new class of strongly demicontractive multi-valued mappings in Hilbert spaces. Strong convergence theorems of Picard and Mann iterative methods for strongly demicontractive multi-valued mappings are established under some suitable coefficients and control sequences.

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