

Dedicated to Prof. Billy E. Rhoades on the occasion of his 90th anniversary

Frum-Ketkov operators which are weakly Picard

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ABSTRACT.

Let (M, d) be a metric space, $X \subset M$ be a nonempty closed subset and $K \subset M$ be a nonempty compact subset. By definition, a continuous operator $f : X \rightarrow X$ is said to be a Frum-Ketkov operator if there exists $l \in]0, 1[$ such that $d(f(x), K) \leq ld(x, K)$, for every $x \in X$. In this paper, we will give sufficient conditions ensuring that a Frum-Ketkov operator is weakly Picard. Some generalized Frum-Ketkov operators are also studied.

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