

Dedicated to Prof. Billy E. Rhoades on the occasion of his 90th anniversary

On $(\psi, \varphi)^2$ - contractive maps

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ABSTRACT.

In this paper, we introduce the notion of generalized weakly contractive mapping of quadratic type and we prove fixed point results for this type of mapping. We justify our result by suitable examples and show that this mapping satisfies properties P and Q . Among other things, as corollaries we recover Banach and Kannan fixed point theorems.

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