Dedicated to Prof. Billy E. Rhoades on the occasion of his 90th anniversary

Modified two-step extragradient method for solving the pseudomonotone equilibrium programming in a real Hibert space

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Abstract.

The purpose of this paper is to come up with an inertial extragradient method for dealing with a class of pseudomonotone equilibrium problems. This method can be a view as an extension of the paper title "A new two-step proximal algorithm of solving the problem of equilibrium programming" by Lyashko and Semenov et al. (Optimization and Its Applications in Control and Data Sciences: 315—325, 2016). The theorem of weak convergence for solutions of the pseudomonotone equilibrium problems is well-established under standard assumptions placed on cost bifunction in the structure of a real Hilbert spaces. For a numerical experiment, we take up a well-known Nash Cournot equilibrium model of electricity markets to support the well-established convergence results and be adequate to see that our proposed algorithms have a competitive superiority over the time of execution and the number of iterations.

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