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Dedicated to Prof. Billy E. Rhoades on the occasion of his 90^{th} anniversary

A Stackelberg-population competition model via variational inequalities and fixed points

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ABSTRACT.

In this paper, we introduce and study a new Stackelberg-population competition model which captures the desired features of both population games and Stackelberg competition model within the same framework. We obtain some characterization results for the Stackelberg-population equilibrium response set and the Stackelberg-population equilibrium leader set by using the variational inequality technique and Brouwer's fixed point theorem. We also show an existence theorem of Nash equilibrium for Stackelberg-population competition model under some mild conditions. Finally, we give an example to illustrate our main results.

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