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On an improved convergence analysis of a two-step Gauss-Newton type method under generalized Lipschitz conditions

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Abstract.

We present a local convergence analysis of a two-step Gauss-Newton method under the generalized and classical Lipschitz conditions for the first- and second-order derivatives. In contrast to earlier works, we use our new idea using a center average Lipschitz conditions through which, we define a subset of the original domain that also contains the iterates. Then, the remaining average Lipschitz conditions are at least as tight as the corresponding ones in earlier works. This way, we obtain: weaker sufficient convergence criteria, larger radius of convergence, tighter error estimates and more precise information on the location of the solution. These advantages are obtained under the same computational effort, since the new Lipschitz functions are special cases of the ones in earlier works. Finally, we give a numerical example that confirms the theoretical results, and compares favorably to the results from previous works.

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