On the crossing number of join of the wheel on six vertices with the discrete graph

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**Abstract.**

The main aim of the paper is to give the crossing number of join product $W_5 + D_n$ for the wheel $W_5$ on six vertices, and $D_n$ consisting of $n$ isolated vertices. In the proofs, it will be extend the idea of the minimum numbers of crossings between two different subgraphs from the family of subgraphs which do not cross the edges of the graph $W_5$ onto the family of subgraphs that cross the edges of $W_5$ at least twice. Further, we give a conjecture that the crossing number of $W_m + D_n$ is equal to $Z(m + 1)Z(n) + (Z(m) - 1)\left\lfloor \frac{n}{2} \right\rfloor + n$ for $m$ at least three, and where the Zarankiewicz’s number $Z(n) = \left\lfloor \frac{n}{2} \right\rfloor \left\lfloor \frac{n-1}{2} \right\rfloor$ is defined for $n \geq 1$. Recently, our conjecture was proved for the graphs $W_m + D_n$, for any $n = 3, 4, 5$, by Klešč et al., and also for $W_3 + D_n$ and $W_4 + D_n$ due to the result by Klešč, Schröter and by Staš, respectively. Clearly, the main result of the paper confirms the validity of this conjecture for the graph $W_5 + D_n$.

**References**


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