

Trace 1, 2×2 matrices over principal ideal domains are exchange

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ABSTRACT.

We prove that trace 1 matrices over principal ideal domains are exchange and characterize 2×2 exchange matrices over commutative domains. In addition, we emphasize large classes of not exchange 2×2 and 3×3 integral matrices.

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