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On optimality conditions for robust weak sharp solution in uncertain optimizations

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ABSTRACT.

In this paper, we investigate the robust optimization problem involving nonsmooth and nonconvex real-valued functions. We firstly establish a necessary condition for the local robust weak sharp solution of considered problem under a constraint qualification. These optimality conditions are presented in terms of multipliers and Mordukhovich subdifferentials of the related functions. Then, by employing the robust version of the (KKT) condition, and some appropriate generalized convexity conditions, we also obtain some sufficient conditions for the global robust weak sharp solutions of the problem. In addition, some examples are presented for illustrating or supporting the results.

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