CARPATHIAN J. MATH. Volume **36** (2020), No. 3, Pages 453 - 462 Online version at https://www.carpathian.cunbm.utcluj.ro/ Print Edition: ISSN 1584 - 2851; Online Edition: ISSN 1843 - 4401 DOI: https://doi.org/10.37193/CJM.2020.03.13

# **Existence of solutions for a fractional nonlocal boundary value problem**

## **RODICA LUCA**

### Abstract.

We investigate the existence of solutions for a Riemann-Liouville fractional differential equation with a nonlinearity dependent of fractional integrals, subject to nonlocal boundary conditions which contain various fractional derivatives and Riemann-Stieltjes integrals. In the proof of our main results we use different fixed point theorems.

**Acknowledgement.** The author thanks the referee for his/her valuable comments and suggestions.

### REFERENCES

- Agarwal, R. P. and Luca, R., Positive solutions for a semipositone singular Riemann-Liouville fractional differential problem, Inter. J. Nonlinear Sci. Num. Simul., 20 (2019), (7-8), 823–832
- [2] Ahmad, B. and Luca, R., Existence of solutions for a sequential fractional integro-differential system with coupled integral boundary conditions, Chaos Solitons Fractals, 104 (2017), 378–388
- [3] Ahmad, B. and Luca, R., Existence of solutions for a system of fractional differential equations with coupled nonlocal boundary conditions, Fract. Calc. Appl. Anal., **21** (2018), No. 2, 423–441
- [4] Ahmad, B. and Luca, R., Existence of solutions for sequential fractional integro-differential equations and inclusions with nonlocal boundary conditions, Appl. Math. Comput., 339 (2018), 516–534
- [5] Ahmad B. and Ntouyas, S. K., Existence results for a coupled system of Caputo type sequential fractional differential equations with nonlocal integral boundary conditions, Appl. Math. Comput., 266 (2015), 615–622
- [6] Aljoudi, S., Ahmad, B, Nieto, J. J. and Alsaedi, A., A coupled system of Hadamard type sequential fractional differential equations with coupled strip conditions, Chaos Solitons Fractals, 91 (2016), 39–46
- [7] Guo, L., Liu, L. and Wu, Y., Iterative unique positive solutions for singular p-Laplacian fractional differential equation system with several parameters, Nonlinear Anal. Model. Control, 23 (2018), No. 2, 182–203
- [8] Henderson, J. and Luca, R., Boundary Value Problems for Systems of Differential, Difference and Fractional Equations. Positive solutions, Elsevier, Amsterdam, 2016
- [9] Henderson, J. and Luca, R., *Existence of positive solutions for a singular fractional boundary value problem*, Nonlinear Anal. Model. Control, **22** (2017), No. 1, 99–114
- [10] Henderson, J. and Luca, R., Systems of Riemann-Liouville fractional equations with multi-point boundary conditions, Appl. Math. Comput., 309 (2017), 303–323
- [11] Henderson, J., Luca, R. and Tudorache, A., On a system of fractional differential equations with coupled integral boundary conditions, Fract. Calc. Appl. Anal., 18 (2015), No. 2, 361–386
- [12] Henderson, J., Luca, R. and Tudorache, A., Existence and nonexistence of positive solutions for coupled Riemann-Liouville fractional boundary value problems, Discrete Dyn. Nature Soc., 2016 Article ID 2823971 (2016), 1–12
- [13] Krasnosel'skii, M. A., Two remarks on the method of successive approximations, Uspekhi Mat. Nauk. 10 (1955), 123–127

Received: 19.04.2019; In revised form: 31.01.2020; Accepted: 07.02.2020 2010 Mathematics Subject Classification. 34A08, 45G15.

Key words and phrases. Riemann-Liouville fractional differential equation, fractional integrals, nonlocal boundary conditions, existence of solutions.

#### Rodica Luca

- [14] Liu, L., Li, H., Liu, C. and Wu, Y., Existence and uniqueness of positive solutions for singular fractional differential systems with coupled integral boundary value problems, J. Nonlinear Sci. Appl., 10 (2017), 243–262
- [15] Liu, S., Liu, J., Dai, Q. and Li, H., Uniqueness results for nonlinear fractional differential equations with infinitepoint integral boundary conditions, J. Nonlinear Sci. Appl., 10 (2017), 1281–1288
- [16] Luca, R., Positive solutions for a system of Riemann-Liouville fractional differential equations with multi-point fractional boundary conditions, Bound. Value Prob., 2017 (2017), No. 102, 1–35
- [17] Pu, R., Zhang, X., Cui, Y., Li, P. and Wang, W., Positive solutions for singular semipositone fractional differential equation subject to multipoint boundary conditions, J. Funct. Spaces, 2017, Article ID 5892616 (2017), 1–7
- [18] Shen, C., Zhou, H. and Yang, L., Positive solution of a system of integral equations with applications to boundary value problems of differential equations, Adv. Difference Equ., 2016 (2016), No. 260, 1–26
- [19] Xu, J. and Wei, Z., Positive solutions for a class of fractional boundary value problems, Nonlinear Anal. Model. Control, 21 (2016), 1–17
- [20] Zhang, X., Positive solutions for a class of singular fractional differential equation with infinite-point boundary conditions, Appl. Math. Lett., 39 (2015), 22–27
- [21] Zhang, X. and Zhong, Q., Triple positive solutions for nonlocal fractional differential equations with singularities both on time and space variables, Appl. Math. Lett., 80 (2018), 12–19

DEPARTMENT OF MATHEMATICS GH. ASACHI TECHNICAL UNIVERSITY 11 BLVD. CAROL I, 700506, IAŞI, ROMANIA *Email address*: rluca@math.tuiasi.ro