An iterative regularization method for variational inequalities in Hilbert spaces

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ABSTRACT.
We consider an iterative method for regularization of a variational inequality (VI) defined by a Lipschitz continuous monotone operator in the case where the set of feasible solutions is decomposed to the intersection of finitely many closed convex subsets of a Hilbert space. We prove the strong convergence of the sequence generated by our algorithm. It seems that this is the first time in the literature to handle iterative solution of ill-posed VIs in the domain decomposition case.

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