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Dedicated to Prof. Ioan A. Rus on the occasion of his 85^{th} anniversary

Ulam-Hyers stability of Darboux-Ionescu problem

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ABSTRACT. In his doctoral thesis, D. V. Ionescu has considered Darboux problem for partial differential equations of order two with modified argument. The Darboux-Ionescu problem was studied in some general cases by I. A. Rus. In this paper we study Ulam-Hyers stability and Ulam-Hyers-Rassias stability for this problem considered by I. A. Rus, using inequalities of Wendorff type.

1. INTRODUCTION

In the paper [7], pg. 40, D. V. Ionescu has studied Darboux, Cauchy, Picard and Goursat problems for hyperbolic partial differential equations of order two with modified argument for functions of two variables. The Darboux-Ionescu problem was studied in a more general frame by I. A. Rus. In 1979, in [16] and in 1981, in [17], [18], I. A. Rus has considered a generalization of the equation (2) from [7]. For this, he has studied the Darboux problem and has formulated an existence, uniqueness and data dependence theorem. This equation was also studied among others by V. Berinde [1] and G. Dezső in [3]. V. Berinde has used generalized Lipschitz conditions and Dezső has studied systems of partial differential equations of hyperbolic type. Equations with modified argument were also studied by D. Otrocol and V. A. Ilea in [6], [11], [12]. Ulam stability of a nonlinear hyperbolic partial differential equation was studied in [20].

In this paper we study Ulam-Hyers stability and Ulam-Hyers-Rassias stability for a generalization of Darboux-Ionescu problem from [18], corresponding to a hyperbolic partial differential equation of order three, using inequalities of Wendorff type.

Let $a,b,c \in (0,\infty)$, $I_3 = [0,a] \times [0,b] \times [0,c]$, $F \in C(I_3 \times \mathbb{R}, \mathbb{R})$, $f \in C(I_3,[0,a]), g \in C(I_3,[0,b]), h \in C(I_3,[0,c])$. Let $\varphi \in C^1([0,a]\times[0,b],\mathbb{R}), \psi \in C^1([0,a]\times[0,c],\mathbb{R}), \chi \in C^1([0,b]\times[0,c],\mathbb{R}), w^1 \in C([0,a],\mathbb{R}), w^2 \in C([0,b],\mathbb{R}), w^3 \in C([0,c],\mathbb{R}), w^0 \in \mathbb{R}$.

In what follows we consider the equation

$$u_{xyz}(x,y,z) = F(x,y,z,u(f(x,y,z),g(x,y,z),h(x,y,z))) \tag{1.1}$$

and the following Darboux problem, analogously as in [18]:

$$\begin{cases} u(x,y,0) = \varphi(x,y), (x,y) \in [0,a]\times[0,b], \\ u(x,0,z) = \psi(x,z), (x,z) \in [0,a]\times[0,c], \\ u(0,y,z) = \chi(y,z), (y,z) \in [0,b]\times[0,c], \end{cases} \tag{1.2}$$

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where φ, ψ, χ satisfy the conditions:

$$\begin{cases} \varphi(x,0) = \psi(0,x) = w^1(x), x \in [0,a], \\ \chi(y,0) = \varphi(0,y) = w^2(y), y \in [0,b], \\ \psi(z,0) = \chi(0,z) = w^3(z), z \in [0,c], \\ w^1(0) = w^2(0) = w^3(0) = w^0. \end{cases} \tag{1.3}$$

We recall first the definition of Picard operator and abstract Gronwall lemma. Let X be a nonemty set. Let $F_A = \{x \in X \mid A(x) = x\}$ be the fixed points set of $A : X \longrightarrow X$. Let $A_0 := 1_X$, $A^1 := A, ..., A^{n+1} := A \circ A^n$, $n \in \mathbb{N}$.

Definition 1.1. ([4]). Let X be a nonempty set. Let $s(X) := \{(x_n)_{n\in\mathbb{N}} | x_n \in X, n \in \mathbb{N}\}$. Let $c(X)$ be a subset of $s(X)$ and $Lim : c(X) \to X$ be an operator. The triple $(X, c(X), Lim)$ is called an L-space (denoted by $(X, \to)$) if the following conditions are satisfied:

(i) if $x_n = x$ for all $n \in \mathbb{N}$, then $(x_n)_{n\in\mathbb{N}} \in c(X)$ and $Lim(x_n)_{n\in\mathbb{N}} = x$;

(ii) if $(x_n)_{n\in\mathbb{N}} \in c(X)$ and $Lim(x_n)_{n\in\mathbb{N}} = x$, then for all subsequences $(x_{n_i})_{i\in\mathbb{N}}$ of $(x_n)_{n\in\mathbb{N}}$ we have that $(x_{n_i})_{i\in\mathbb{N}} \in c(X)$ and $Lim(x_{n_i})_{i\in\mathbb{N}} = x$.

An element of $c(X)$ is called a convergent sequence and $x = Lim(x_n)_{n\in\mathbb{N}}$ is the limit of this sequence. We write $(x_n)_{n\in\mathbb{N}} \to x$ or $x_n \to x$ as $n \to \infty$.

Definition 1.2. ([14]). Let X be a nonempty set. $(X, \to, \leq)$ is called an ordered L-space if:

(i) $(X, \to)$ is an L-space;

(ii) $(X, \leq)$ is a partially ordered set;

(iii) if $(x_n)_{n\in\mathbb{N}} \to x$, $(y_n)_{n\in\mathbb{N}} \to y$ and $x_n \leq y_n$ for each $n \in \mathbb{N}$, then $x \leq y$.

Definition 1.3. ([19]). Let $(X, \to)$ be an L-space. An operator $A : X \longrightarrow X$ is called a Picard operator if there exists $x_A^* \in X$ such that $F_A = \{x_A^*\}$ and $A^n(x) \longrightarrow x_A^*$ as $n \longrightarrow \infty$, for all $x \in X$.

Lemma 1.1. (Abstract Gronwall Lemma [2], [15], [21]) (AGL). *Let $(X, \to, \leq)$ be an ordered $L-$space and $A : X \to X$ an operator. We suppose that:*

(i) *A is a Picard operator;*

(ii) *A is an increasing operator.*

If we denote by x_A^* the unique fixed point of A, then we have:

$$x \in X,\ x \leq A(x) \Rightarrow x \leq x_A^*. \tag{1.4}$$

2. Ulam-Hyers stability

For a better understanding we recall some notions and definitions.

Let $a, b, c \in (0, \infty), \varepsilon > 0$, $I_3 = [0,a] \times [0,b] \times [0,c]$, $k \in C(I_3, \mathbb{R}_+)$ and the inequalities

$$|v_{xyz}(x,y,z) - F(x,y,z,v(f(x,y,z), g(x,y,z), h(x,y,z)))| \leq \varepsilon, (x,y,z) \in I_3, \tag{2.5}$$

$$|v_{xyz}(x,y,z) - F(x,y,z,v(f(x,y,z), g(x,y,z), h(x,y,z)))| \leq k(x,y,z), (x,y,z) \in I_3. \tag{2.6}$$

Definition 2.4. The equation (1.1) is Ulam-Hyers stable if there exists a real number $c_F > 0$ such that for each solution v of the inequality (2.5), there exists a solution u of the equation (1.1) with

$$|v(x,y,z) - u(x,y,z)| \leq c_F \cdot \varepsilon, \forall (x,y,z) \in I_3. \tag{2.7}$$

Definition 2.5. The equation (1.1) is Ulam-Hyers-Rassias stable if there exists a real number $c_{Fk} > 0$ such that for each solution v of the inequality (2.6), there exists a solution u of the equation (1.1) with

$$|v(x,y,z) - u(x,y,z)| \leq c_{Fk} \cdot k(x,y,z), \forall (x,y,z) \in I_3. \tag{2.8}$$

Remark 2.1. A function v is a solution of (2.5) if and only if there exists a function $p \in C(I_3, \mathbb{R})$ such that

(i) $|p(x,y,z)| \le \varepsilon, \forall (x,y,z) \in I_3$,
(ii) $v_{xyz}(x,y,z) = F(x,y,z,v(f(x,y,z), g(x,y,z), h(x,y,z))) + p(x,y,z), \forall (x,y,z) \in I_3$.

From this remark it follows:

Corollary 2.1. *If v is a solution of (2.5), then v is a solution of the integral inequality*

$$\begin{aligned}(2.9)\quad &\Big|v(x,y,z) - \varphi(x,y) - \psi(z,x) - \chi(y,z) + w^1(x) + w^2(y) + w^3(z) - w^0 \\ &- \int_0^x \int_0^y \int_0^z F(r,s,t,v(f(r,s,t), g(r,s,t), h(r,s,t)))\,drdsdt\Big| \le \varepsilon xyz,\end{aligned}$$

$\forall (x,y,z) \in I_3$.

Proof. Indeed, from (1.2), (1.3) and Remark 2.1 we have

$$\begin{aligned}v(x,y,z) &= \varphi(x,y) + \psi(z,x) + \chi(y,z) - w^1(x) - w^2(y) - w^3(z) + w^0 \\ &+ \int_0^x \int_0^y \int_0^z F(r,s,t,v(f(r,s,t), g(r,s,t), h(r,s,t)))\,drdsdt + \int_0^x \int_0^y \int_0^z p(r,s,t)\,drdsdt.\end{aligned}$$

Hence the inequality (2.9) is satisfied. □

Analogously, we have the following remark and consequence.

Remark 2.2. A function v is a solution of (2.6) if and only if there exists a function $p \in C(I_3, \mathbb{R})$ such that

(i) $|p(x,y,z)| \le k(x,y,z), \forall (x,y,z) \in I_3$,
(ii) $v_{xyz}(x,y,z) = F(x,y,z,v(f(x,y,z), g(x,y,z), h(x,y,z))) + p(x,y,z), \forall (x,y,z) \in I_3$.

Corollary 2.2. *If v is a solution of (2.6), then v is a solution of the integral inequality*

$$\begin{aligned}(2.10)\quad &\Big|v(x,y,z) - \varphi(x,y) - \psi(z,x) - \chi(y,z) + w^1(x) + w^2(y) + w^3(z) - w^0 \\ &- \int_0^x \int_0^y \int_0^z F(r,s,t,v(f(r,s,t), g(r,s,t), h(r,s,t)))\,drdsdt\Big| \\ &\le \int_0^x \int_0^y \int_0^z k(r,s,t)\,drdsdt, \forall (x,y,z) \in I_3\end{aligned}$$

Theorem 2.1. *We suppose that the following conditions are satisfied:*

(i) $F \in C(I_3 \times \mathbb{R}, \mathbb{R})$, $f \in C(I_3, [0,a]), g \in C(I_3, [0,b]), h \in C(I_3, [0,c])$, $f(x,y,z) \le x, g(x,y,z) \le y, h(x,y,z) \le z, \forall (x,y,z) \in I_3$;
(ii) $\varphi \in C^1([0,a] \times [0,b], \mathbb{R}), \psi \in C^1([0,a] \times [0,c], \mathbb{R}), \chi \in C^1([0,b] \times [0,c], \mathbb{R})$, $w^1 \in C([0,a], \mathbb{R}), w^2 \in C([0,b], \mathbb{R}), w^3 \in C([0,c], \mathbb{R}), w^0 \in \mathbb{R}$ *verify the conditions* (1.3);
(iii) $\exists L_F > 0$ *such that*
$$|F(x,y,z,u_1) - F(x,y,z,u_2)| \le L_F |u_1 - u_2|, \forall (x,y,z) \in I_3, u_1, u_2 \in \mathbb{R};$$
(iv) $\exists \tau > 0$ *and* $0 < q < 1$ *such that*
$$\int_0^x \int_0^y \int_0^z L_F e^{\tau[f(r,s,t)+g(r,s,t)+h(r,s,t)-x-y-z]}\,drdsdt \le q, \forall (x,y,z) \in I_3;$$
(v) $L_F abc < 1$.

Then:

(a) *the problem* (1.1)+(1.2)+(1.3) *has a unique solution;*
(b) *the equation* (1.1) *is Ulam-Hyers stable.*

Proof. (a) This is a known existence and uniqueness result (see Theorem 3.1, pg. 12, [18]).
(b) Let v be a solution of (2.5) and let u be the unique solution of the problem (1.1)+(1.2)+(1.3). Inequality (2.9) and condition (iii) imply

$$|v(x,y,z)-u(x,y,z)| \leq \Big|v(x,y,z)-\varphi(x,y)-\psi(z,x)-\chi(y,z)+w^1(x)+w^2(y)+w^3(z)$$
$$-w^0-\int_0^x\int_0^y\int_0^z F(r,s,t,v(f(r,s,t),g(r,s,t),h(r,s,t)))\,drdsdt\Big|$$
$$+\int_0^x\int_0^y\int_0^z |F(r,s,t,v(f(r,s,t),g(r,s,t),h(r,s,t)))-F(r,s,t,u(f(r,s,t),g(r,s,t),h(r,s,t)))|drdsdt$$
$$\leq \varepsilon xyz+L_F\int_0^x\int_0^y\int_0^z |v(f(r,s,t),g(r,s,t),h(r,s,t))-u(f(r,s,t),g(r,s,t),h(r,s,t))|drdsdt.$$

We consider the operator $A: C(I_3,\mathbb{R}_+)\to C(I_3,\mathbb{R}_+)$,

$$A(w)(x,y,z)=\varepsilon xyz+L_F\int_0^x\int_0^y\int_0^z w(f(r,s,t),g(r,s,t),h(r,s,t))drdsdt, \forall (x,y,z)\in I_3.$$

We verify that A is a Picard operator. We prove that A is a contraction. We have:

$$|A(w_1)(x,y,z)-A(w_2)(x,y,z)|$$
$$\leq L_F\int_0^x\int_0^y\int_0^z |w_1(f(r,s,t),g(r,s,t),h(r,s,t))-w_2(f(r,s,t),g(r,s,t),h(r,s,t))|drdsdt$$
$$\leq L_F\int_0^x\int_0^y\int_0^z \max_{(r,s,t)\in I_3}|w_1(r,s,t)-w_2(r,s,t)|drdsdt$$
$$\leq L_F abc\,\|w_1-w_2\|.$$

Hence

$$\|A(w_1)-A(w_2)\|\leq L_F abc\,\|w_1-w_2\|.$$

Using condition (v) we obtain that A is a contraction with respect to Chebyshev norm on I_3. Applying Banach contraction principle we have that A is Picard operator and $F_A=\{w^*\}$.

Hence

$$w^*(x,y,z)=\varepsilon xyz+L_F\int_0^x\int_0^y\int_0^z w^*(f(r,s,t),g(r,s,t),h(r,s,t))\,drdsdt, \forall (x,y,z)\in I_3.$$

The solution w^* is increasing, $w^*(f(r,s,t),g(r,s,t),h(r,s,t))\leq w^*(r,s,t)$, so

$$w^*(x,y,z)\leq \varepsilon xyz+L_F\int_0^x\int_0^y\int_0^z w^*(r,s,t)\,drdsdt, \forall (x,y,z)\in I_3.$$

From Wendorff inequality in three dimensions (see [5], [8], [9], [10], [13]) we obtain

$$w^*(x,y,z)\leq \varepsilon abc\exp(L_F abc).$$

We now consider $w:=|v-u|$. From

$$w(x,y,z)\leq A(w)(x,y,z)$$

and applying abstract Gronwall lemma we obtain

$$w(x,y,z)\leq w^*(x,y,z), \forall (x,y,z)\in I_3,$$

since A is a Picard and increasing operator. We have

$$|v(x,y,z)-u(x,y,z)|\leq \varepsilon abc\exp(L_F abc), \forall (x,y,z)\in I_3,$$

hence the equation is Ulam-Hyers stable.

□

Remark 2.3. If $a = b = c = \infty$, then the equation is not Ulam-Hyers stable. For example let $\varphi \in C^1([0,\infty) \times [0,\infty), \mathbb{R}), \psi \in C^1([0,\infty) \times [0,\infty), \mathbb{R}), \chi \in C^1([0,\infty) \times [0,\infty), \mathbb{R})$, $w^1 \in C([0,\infty), \mathbb{R})$, $w^2 \in C([0,\infty), \mathbb{R})$, $w^3 \in C([0,\infty), \mathbb{R})$, $w^0 = 0$. We consider the homogeneous equation $u_{xyz}(x,y,z) = 0$. In this case $u(x,y,z) = u_1(x) + u_2(y) + u_3(z)$, $u_1(0) = u_2(0) = u_3(0) = 0$ is a solution of the equation (1.1). Let

$$\left\{ \begin{array}{l} u(x,y,0) = \varphi(x,y) = u_1(x) + u_2(y), (x,y) \in [0,\infty) \times [0,\infty), \\ u(x,0,z) = \psi(x,z) = u_1(x) + u_3(z), (x,z) \in [0,\infty) \times [0,\infty), \\ u(0,y,z) = \chi(y,z) = u_2(y) + u_3(z), (y,z) \in [0,\infty) \times [0,\infty), \end{array} \right. \tag{2.11}$$

and

$$\left\{ \begin{array}{l} \varphi(x,0) = \psi(0,x) = w^1(x) = u_1(x), x \in [0,\infty) \\ \chi(y,0) = \varphi(0,y) = w^2(y) = u_2(y), y \in [0,\infty) \\ \psi(z,0) = \chi(0,z) = w^3(z) = u_3(z), z \in [0,\infty) \\ w^1(0) = w^2(0) = w^3(0) = w^0 = 0. \end{array} \right. \tag{2.12}$$

We choose the solution $v(x,y,z) = \varepsilon xyz + u_1(x) + u_2(y) + u_3(z)$ of inequality (2.9). In this case $|v(x,y,z) - u(x,y,z)| = |\varepsilon xyz|$ is unbounded when x, y, z tend to ∞.

Example 2.1. Let $I_3 = [0,1] \times [0,1] \times [0,1]$, $f \in C(I_3, [0,1]), g \in C(I_3, [0,1]), h \in C(I_3, [0,1])$, $f(x,y,z) = x, g(x,y,z) = y^2, h(x,y,z) = z^3$.

We consider $F \in C([0,1] \times [0,1] \times [0,1] \times \mathbb{R}, \mathbb{R}), F(x,y,z,u) = \frac{1}{4}xyzu^2$. The corresponding Lipschitz constant with respect to u is $\frac{1}{2}$.

We suppose that the conditions (ii) from from Theorem 2.1 are satisfied. We remark that all the conditions from Theorem 2.1 are satisfied, hence the equation is Ulam-Hyers stable.

3. Ulam-Hyers-Rassias stability

We study now Ulam-Hyers-Rassias stability of the equation (1.1).

Theorem 3.2. *We suppose that the following conditions are satisfied:*

(i) $F \in C(I_3 \times \mathbb{R}, \mathbb{R})$, $f \in C(I_3, [0,a]), g \in C(I_3, [0,b]), h \in C(I_3, [0,c])$, $f(x,y,z) \leq x, g(x,y,z) \leq y, h(x,y,z) \leq z, \forall (x,y,z) \in I_3$;

(ii) $\varphi \in C^1([0,a] \times [0,b], \mathbb{R}), \psi \in C^1([0,a] \times [0,c], \mathbb{R}), \chi \in C^1([0,b] \times [0,c], \mathbb{R})$, $w^1 \in C([0,a], \mathbb{R}), w^2 \in C([0,b], \mathbb{R}), w^3 \in C([0,c], \mathbb{R}), w^0 \in \mathbb{R}$ *verify the conditions* (1.3);

(iii) $\exists L_F \in C(I_3, \mathbb{R}_+)$ *such that*

$$|F(x,y,z,u_1) - F(x,y,z,u_2)| \leq L_F(x,y,z)|u_1 - u_2|, \forall (x,y,z) \in I_3, u_1, u_2 \in \mathbb{R};$$

(iv) $\exists \tau > 0$ *and* $0 < q < 1$ *such that*

$$\int_0^x \int_0^y \int_0^z L_F(r,s,t) e^{\tau[f(r,s,t)+g(r,s,t)+h(r,s,t)-x-y-z]} drdsdt \leq q, \forall (x,y,z) \in I_3;$$

(v) $k \in C(I_3, \mathbb{R}_+)$ *is increasing;*

(vi $\exists \lambda_k > 0$ *such that*

$$\int_0^x \int_0^y \int_0^z k(x,y,z) drdsdt \leq \lambda_k k(x,y,z), \forall x,y,z \in [0,\infty);$$

(vii) *There exists a real number M such that* $e^{\int_0^x \int_0^y \int_0^z L_F(r,s,t)drdsdt} \leq M, \forall (x,y,z) \in I_3$.

Then:

(a) *the problem* (1.1)+(1.2)+(1.3) *has a unique solution;*

(b) *the equation* (1.1) *is Ulam-Hyers-Rassias stable.*

Proof. Let v be a solution of (2.6) and let u be the unique solution of the corresponding Darboux problem. From inequality (2.10) and condition (iii) we have

$$|v(x,y,z)-u(x,y,z)| \leq \lambda_k k(x,y,z)$$
$$+\int_0^x\int_0^y\int_0^z L_F(r,s,t)\,|v(f(r,s,t),g(r,s,t),h(r,s,t)) - u(f(r,s,t),g(r,s,t),h(r,s,t))|drdsdt.$$

Using Wendorff lemma for three dimensions and conditions (vi), (vii) we obtain

$$|v(x,y,z)-u(x,y,z)| \leq \left[\lambda_k e^{\int_0^x\int_0^y\int_0^z L_F(r,s,t)drdsdt}\right] k(x,y,z) \leq c_{Fk} k(x,y,z),$$

$\forall x,y,z \in [0,\infty)$, where $c_{Fk} = \lambda_k M$. Hence the equation is Ulam-Hyers-Rassias stable. □

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