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Dedicated to Prof. Emeritus Mihail Megan on the occasion of his 75^{th} anniversary

A case study in set-mapping pair theory: the set-mapping pair bijections

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ABSTRACT. In a recent paper (I. A. Rus. Sets with structure, mappings and fixed point property: fixed point structures. *Fixed Point Theory* **23** (2022), No. 2) the author introduced, amongst others, the following notions: set-mapping pair and set-mapping pair bijection. In this paper we study the set-mapping pair bijections in connection with the isomorphisms structure, and their impact on the fixed point theory in a set-mapping pair. Some open problems are also formulated.

1. INTRODUCTION AND PRELIMINARIES

Let X and Y be two nonempty sets with some structure (ordered sets, or metric spaces, or L-spaces, or topological spaces, or Kasahara spaces, or compact subsets of a topological space, or convex subsets of a Banach space, ...), as in [18] and the references therein (J. Dieudonné (1982), S. Mac Lane (1996), K. Denecke (2007), D. Duffus and I. Rival (1981), D. Gorenstein (1968), G. Longo (1983), B. Schröder (2003), S. Vasilache (1956), ...).

We denote by $\mathbb{M}(X,Y) := \{f : X \to Y \mid f$ is a mapping from the set X to the set $Y\}$ and by $Hom(X,Y) := \{f : X \to Y \mid f$ is a mapping which preserves the structure, i.e., a morphism$\}$.

The following notions were introduced in [18].

Definition 1.1. Let $\mathcal{U}$ be a class of nonempty sets with some structure. We suppose that for each ordered pair (X,Y) with X and Y in $\mathcal{U}$, a set of mappings from the set X to the set Y, $M(X,Y)$ is given. By $f \in M$ we understand that there exist X and Y in $\mathcal{U}$ such that, $f \in M(X,Y)$. By definition, the pair $(\mathcal{U}, M)$ is a set-mapping pair.

Definition 1.2. A pair $(\mathcal{U}, M)$ is with composition if for any $f, g \in M$ such that $f \circ g$ is defined, we have $f \circ g \in M$.

Definition 1.3. A pair $(\mathcal{U}, M)$ is with identity if it is with composition and for each $X \in \mathcal{U}$, the identity mapping belongs to $(\mathcal{U}, M)$, i.e., $1_X \in M(X,X)$.

Definition 1.4. Let $(\mathcal{U}, M)$ be a set-mapping pair. By definition, a bijective mapping $f \in \mathbb{M}(X,Y)$, with $X, Y \in \mathcal{U}$, is a $(\mathcal{U}, M)$-bijection if for all $g \in M(X,X)$ and $h \in M(Y,Y)$ we have that, $f^{-1} \circ h \circ f \in M(X,X)$ and $f \circ g \circ f^{-1} \in M(Y,Y)$. We also call such a bijective mapping, a set-mapping pair bijection.

For examples and counterexamples related to these notions see [18]. Regarding the notion of isomorphism in a category see [2], [11], [3], [4], [13].

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The aim of this paper is to study the set-mapping pair bijections in connection with the isomorphisms structure and its impact on the fixed point theory in a set-mapping pair.

The structure of the paper is the following:

1. Introduction and preliminaries
2. Isomorphisms in $(\mathcal{U}, Hom)$ and $(\mathcal{U}, M)$-bijections
3. Properties preserved by $(\mathcal{U}, M)$-bijections
4. Set-mapping pairs in terms of Kasahara spaces
5. Conclusions

Throughout this paper we follow the notations and terminology given in [18].

2. Isomorphisms in $(\mathcal{U}, Hom)$ and $(\mathcal{U}, M)$-bijections

Let $\mathcal{C}$ be a category of sets with some structure, i.e., the class of objects of $\mathcal{C}$. Let $\mathcal{U}$ be the class of sets having the same structure as the sets belonging to $\mathcal{C}$. Let $Hom(X,Y) := \{f : X \to Y \mid f$ preserves the structure$\}$. The class of Hom is a partial monoid with respect to composition. By definition, for $X, Y \in \mathcal{U}$, an element $\varphi \in Hom(X,Y)$ is an isomorphism if φ is a bijection and $\varphi^{-1} \in Hom(Y,X)$. If for two objects of $\mathcal{C}$, i.e. two sets $X, Y \in \mathcal{U}$, there exists an isomorphism, then we call X and Y isomorphic. It is clear that, $(\mathcal{U}, Hom)$ is a set-mapping pair.

Example 2.1 (The category *Ord*)**.** In this case, $\mathcal{U} :=$ the class of all ordered sets (for the notations and terminology in ordered sets see [12]) and $Hom(X,Y) := \{f : X \to Y \mid f$ is increasing$\}$. A bijection $\varphi \in Hom(X,Y)$ is an isomorphism iff $\varphi^{-1} \in Hom(Y,X)$, i.e., φ and φ^{-1} are increasing. If φ is an isomorphism in Ord, then φ is a $(\mathcal{U}, Hom)$-bijection.

Example 2.2 (The category *Met*)**.** In this case, $\mathcal{U} :=$ the class of all metric spaces and for X and Y metric spaces, $Hom(X,Y) := \{f : X \to Y \mid f$ is nonexpansive$\}$. An element $\varphi \in Hom(X,Y)$ is an isomorphism iff φ is an isometry, i.e., $\varphi : (X, d_X) \to (Y, d_Y)$ is a bijection such that, $d_Y(\varphi(x), \varphi(y)) = d_X(x,y)$, for all $x, y \in X$.

We observe that each isometry is a $(\mathcal{U}, Hom)$-bijection. Moreover, if $\varphi : (X, d_X) \to (Y, d_Y)$ is a bijection such that there exists $k > 0$, for which we have that

$$d_Y(\varphi(x), \varphi(y)) = k d_X(x,y),\ \forall\, x, y \in X,$$

then φ is a $(\mathcal{U}, Hom)$-bijection, which, in general, is not in Hom. Indeed, let us consider for example $g \in Hom(X,X)$, i.e., $d_X(g(x), g(y)) \leq d_X(x,y)$, $\forall\, x, y \in X$. We have that,

$$\begin{aligned} &d_Y(\varphi(g(\varphi^{-1}(x))), \varphi(g(\varphi^{-1}(y)))) = k d_X(g(\varphi^{-1}(x), g(\varphi^{-1}(y))) \\ &\qquad \leq k d_X(\varphi^{-1}(x), \varphi^{-1}(y)) \leq d_Y(x,y),\ \forall\, x, y \in Y, \text{ i.e. } \varphi \circ g \circ \varphi^{-1} \in Hom(Y,Y). \end{aligned}$$

Example 2.3 (The category *HTop*)**.** In this case $\mathcal{U} :=$ the class of all Hausdorff topological spaces and $Hom(X,Y) := C(X,Y) := \{f : X \to Y \mid f$ is continuous$\}$. A bijection $\varphi : X \to Y$ is an isomorphism if φ and φ^{-1} are continuous. Each isomorphism in $HTop$ is a $(\mathcal{U}, Hom)$-bijection.

Example 2.4. Let $\mathcal{U} :=$ the class of all ordered sets and $M(X,Y) := \{f : X \to Y \mid f$ is decreasing$\}$. Then $(\mathcal{U}, M)$ is a set-mapping pair. This pair is not with composition. If $\varphi : X \to Y$ is a bijection such that φ and φ^{-1} are increasing, then φ is a $(\mathcal{U}, M)$-bijection. Indeed, let for example, $g : X \to X$ be decreasing, $x, y \in Y$, $x \leq y$. We have that, $\varphi^{-1}(x) \leq \varphi^{-1}(y)$, $g(\varphi^{-1}(x)) \geq g(\varphi^{-1}(y))$, $\varphi(g(\varphi^{-1}(x))) \geq \varphi(g(\varphi^{-1}(y)))$, i.e., $\varphi \circ g \circ \varphi^{-1} \in M(Y,Y)$.

Example 2.5. Let $(X, \leq)$ be an ordered set. We take $\mathcal{U} := \{(Y, \leq) \mid Y \subset X,\ Y \neq \varnothing\}$ and for $Y, Z \in \mathcal{U}$, $M(Y,Z) := \{f : Y \to Z \mid f$ is progressive, i.e., $x \leq f(x),\ \forall\, x \in Y\}$. If $\varphi : Y \to Z$ is a bijection such that φ and φ^{-1} are increasing, then φ is a $(\mathcal{U}, M)$-bijection. Indeed, for example, if $g \in M(Y,Y)$ and $z \in Z$, then $\varphi^{-1}(z) \leq g(\varphi^{-1}(z))$ and $z \leq \varphi(g(\varphi^{-1}(z)))$, i.e., $\varphi \circ g \circ \varphi^{-1} \in M(Z,Z)$.

From the above examples, we have the following open problems.

Problem 2.1. *In which category $(\mathcal{U}, Hom)$ of sets with structure, each $(\mathcal{U}, Hom)$-bijection is an isomorphism?*

Problem 2.2. *If a category is not a solution of Problem 2.1, the problem is to give examples of $(\mathcal{U}, Hom)$-bijections which are not isomorphisms in $(\mathcal{U}, Hom)$.*

Remark 2.1. If φ and ψ are isomorphisms in a category of sets with structure such that $\psi \circ \varphi$ is defined, then $\psi \circ \varphi$ is an isomorphism. We have a similar property for $(\mathcal{U}, M)$-bijections.

Let $(\mathcal{U}, M)$ be a set-mapping pair and $\varphi, \psi \in M$ be two $(\mathcal{U}, M)$-bijections. If $\psi \circ \varphi$ is defined, then $\psi \circ \varphi$ is a $(\mathcal{U}, M)$-bijection.

Indeed, let $\varphi \in M(X, Y)$ and $\psi \in M(Y, Z)$ and, for example, $h \in M(X, X)$. From $h \in M(X, X)$ we have that, $\varphi \circ h \circ \varphi^{-1} \in M(Y, Y)$, $\psi(\varphi \circ h \circ \varphi^{-1})\psi^{-1} \in M(Z, Z)$, i.e., $(\psi \circ \varphi) \circ h \circ (\psi \circ \varphi)^{-1} \in M(Z, Z)$.

Remark 2.2. More considerations on the set theory and the category theory can be found in: [4], [2], [11], [14], [6], [17], [20], [3].

3. Properties preserved by $(\mathcal{U}, M)$-bijections

The following notions were introduced in [18].

Let $(\mathcal{U}, M)$ be a set-mapping pair and $\mathcal{S} \subset \mathcal{U}$, $\mathcal{S} \neq \varnothing$.

Definition 3.5. The triple $(\mathcal{U}, \mathcal{S}, M)$ is a fixed point structure $(f.p.s.)$ on $(\mathcal{U}, M)$ if for each $X \in \mathcal{S}$ and $f \in M(X, X)$, the fixed point set of f is not empty, i.e., $F_f \neq \varnothing$.

Let $\mathcal{S}_{max} :=$ the class of all $X \in \mathcal{U}$ such that if $f \in M(X, X)$ then $F_f \neq \varnothing$. By definition, the triple $(\mathcal{U}, \mathcal{S}_{max}, M)$ is the maximal f.p.s. on $(\mathcal{U}, M)$.

Definition 3.6. Let $(\mathcal{U}, \mathcal{S}, M)$ be a f.p.s. on $(\mathcal{U}, M)$. An element $X \in \mathcal{S}$ has the common fixed point property if the following implication holds,

$$f, g \in M(X, X),\ f \circ g = g \circ f \ \Rightarrow\ F_f \cap F_g \neq \varnothing.$$

Definition 3.7. Let $(\mathcal{U}, M)$ be a set mapping pair and $(\mathcal{U}, \mathcal{S}, M)$ be a f.p.s. on $(\mathcal{U}, M)$. By definition, an element $X \in \mathcal{S}$ has the coincidence point property if the following implication holds,

$$f, g \in M(X, X),\ f \circ g = g \circ f \ \Rightarrow\ C(f, g) := \{x \in X \mid f(x) = g(x)\} \neq \varnothing.$$

Theorem 3.1. *Let $(\mathcal{U}, M)$ be a set mapping pair, $X \in \mathcal{S}_{max}$ and $Y \in \mathcal{U}$. If there exists a $(\mathcal{U}, M)$-bijection, $\varphi : X \to Y$, then:*

(1) *$Y \in \mathcal{S}_{max}$;*
(2) *if X has the common fixed point property then Y has the common fixed point property;*
(3) *if X has the coincidence point property then Y has the coincidence point property.*

Proof. (1). Let $g \in M(Y, Y)$. Since $\varphi : X \to Y$ is a $(\mathcal{U}, M)$-bijection, it follows that $\varphi^{-1} \circ g \circ \varphi \in M(X, X)$. Since $X \in \mathcal{S}_{max}$, we have $F_{\varphi^{-1} \circ g \circ \varphi} \neq \varnothing$. So, there exists $x^* \in X$ such that $\varphi^{-1}(g(\varphi(x^*))) = x^*$. It follows that $g(\varphi(x^*)) = \varphi(x^*)$, so $F_g \neq \varnothing$. Hence, $Y \in \mathcal{S}_{max}$.

(2). Let $g, h \in M(Y, Y)$ such that $g \circ h = h \circ g$. Since $\varphi : X \to Y$ is a $(\mathcal{U}, M)$-bijection, we have $\varphi^{-1} \circ g \circ \varphi \in M(X, X)$ and $\varphi^{-1} \circ h \circ \varphi \in M(X, X)$. In addition, $\varphi^{-1} \circ g \circ \varphi$ and $\varphi^{-1} \circ h \circ \varphi$ are commuting mappings.

Since X has the common fixed point property, it follows that $F_{\varphi^{-1} \circ g \circ \varphi} \cap F_{\varphi^{-1} \circ h \circ \varphi} \neq \varnothing$. So, there exists $x^* \in X$ such that $x^* = (\varphi^{-1} \circ g \circ \varphi)(x^*)$ and $x^* = (\varphi^{-1} \circ h \circ \varphi)(x^*)$. It follows that $\varphi(x^*) = g(\varphi(x^*)) = h(\varphi(x^*))$. Hence $F_g \cap F_h \neq \varnothing$ and the conclusion follows.

(3). Let $g, h \in M(Y,Y)$ such that $g \circ h = h \circ g$. Since $\varphi : X \to Y$ is a $(\mathcal{U}, M)$-bijection, we have $\varphi^{-1} \circ g \circ \varphi \in M(X,X)$, $\varphi^{-1} \circ h \circ \varphi \in M(X,X)$ and both are commuting mappings.

Since X has the coincidence point property, there exists $x^* \in X$ such that $\varphi^{-1}(g(\varphi(x^*))) = \varphi^{-1}(h(\varphi(x^*)))$. It follows that $g(\varphi(x^*)) = h(\varphi(x^*))$. So, $C(g,h) \neq \varnothing$ and the conclusion follows. □

Remark 3.3. More considerations on the fixed point property, common fixed point property and coincidence point property, can be found in [10], [19], [1], [5], [6], [14], [16], [20], [13].

4. Set-mapping pairs in terms of Kasahara spaces

Let X be a nonempty set. Let $\overset{X}{\to}$ be an L-space structure on X and $d_X : X \times X \to \mathbb{R}_+$ be a metric on X. By definition (see [15], [7], [8], [9]), the triple $(X, \overset{X}{\to}, d_X)$ is called a Kasahara space if:

(1) if $\{x_n\}_{n\in\mathbb{N}} \subset X$ is a Cauchy sequence with respect to d_X, then it is convergent with respect to $\overset{X}{\to}$;

(2) if $x_n \overset{X}{\to} x^*$, $y_n \overset{X}{\to} y^*$ and $d_X(x_n, y_n) \to 0$, then $x^* = y^*$.

If the triple $(X, \overset{X}{\to}, d_X)$ satisfies only the condition (2), then we call it pre-Kasahara space.

The following result is well known (see [15], [7]).

Contraction principle in a Kasahara space. *Let $(X, \overset{X}{\to}, d_X)$ be a Kasahara space and $f : X \to X$ be a mapping. We suppose that:*

(1) *$f : (X, \overset{X}{\to}) \to (X, \overset{X}{\to})$ is continuous;*

(2) *$f : (X, d_X) \to (X, d_X)$ is a contraction.*

Then, $F_f = \{x^*\}$.

Example 4.6. Let X be a nonempty set, d_X and ρ_X be two metrics on X. If there exists $c > 0$ such that, $\rho_X(x,y) \leq c d_X(x,y)$, for all $x, y \in X$, then the triple $(X, \overset{\rho_X}{\to}, d_X)$ is a Kasahara space.

Example 4.7. Let $X := \mathbb{R}$, $c(\mathbb{R}) := c_1(\mathbb{R}) \cup c_2(\mathbb{R}) \cup c_3(\mathbb{R})$, where $c_1(\mathbb{R})$ is the set of all convergent sequences with respect to the metric $d_X : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_+$, defined by $d_X(x,y) = |x - y|$, for all $x, y \in \mathbb{R}$ and, on $c_1(\mathbb{R})$, we consider $\overset{X}{\to} := \overset{d_X}{\to}$; $c_2(\mathbb{R})$ is the set of all subsequences $\{x_n\}_{n\in\mathbb{N}}$ of $\{n\}_{n\in\mathbb{N}}$ with $x_n \overset{X}{\to} 0$; $c_3(\mathbb{R})$ be the set of all subsequences $\{y_n\}_{n\in\mathbb{N}}$ of $\{n + \frac{1}{n+1}\}_{n\in\mathbb{N}}$ with $y_n \overset{X}{\to} 1$. Notice that $(\mathbb{R}, c(\mathbb{R}), \overset{X}{\to})$ is an L-space. But the triple $(\mathbb{R}, \overset{X}{\to}, d_X)$ is not a Kasahara space. The condition (1) of Kasahara space definition is satisfied, but the condition (2) is not. Indeed, let $x_n := n$ and $y_n := n + \frac{1}{n+1}$, for all $n \in \mathbb{N}$. For these two sequences we have $x_n \overset{X}{\to} 0$, $y_n \overset{X}{\to} 1$ and $d_X(x_n, y_n) \to 0$ as $n \to \infty$, but $0 \neq 1$.

Example 4.8. Let $\mathcal{U} :=$ the class of all Kasahara spaces and for $X, Y \in \mathcal{U}$, $Hom(X,Y) := \{f : X \to Y \mid f : (X, d_X) \to (Y, d_Y)$ is nonexpansive and $f : (X, \overset{X}{\to}) \to (Y, \overset{Y}{\to})$ is continuous$\}$. Then $(\mathcal{U}, Hom)$ is a category. A mapping $f : X \to Y$ is an isomorphism iff:

(i) f is a bijection;

(ii) $f : (X, \overset{X}{\to}) \to (Y, \overset{Y}{\to})$ and $f^{-1} : (Y, \overset{Y}{\to}) \to (X, \overset{X}{\to})$ are continuous;

(iii) $f : (X, d_X) \to (Y, d_Y)$ is an isometry.

In this category we have $(\mathcal{U}, Hom)$-bijections which are not isomorphisms (see Example 2.2).

Example 4.9. Let $\mathcal{U} :=$ the class of all Kasahara spaces and for $X, Y \in \mathcal{U}$, $M(X,Y) := \{f : X \to Y \mid f : (X, \overset{X}{\to}) \to (Y, \overset{Y}{\to})$ is continuous and $f : (X, d_X) \to (Y, d_Y)$ is a contraction$\}$. The set-mapping pair $(\mathcal{U}, M)$ is with composition but is not with identity. Let $f \in \mathbb{M}(X,Y)$ be such that:

(a) f is a bijection;

(b) $f : (X, \overset{X}{\to}) \to (Y, \overset{Y}{\to})$ and $f^{-1} : (Y, \overset{Y}{\to}) \to (X, \overset{X}{\to})$ are continuous;

(c) there exists $k > 0$ such that $d_Y(f(x), f(y)) = kd_X(x,y),\ \forall\, x, y \in X.$

Then f is a $(\mathcal{U}, M)$-bijection.

Example 4.10. Let $\mathcal{U} :=$ the class of all pre-Kasahara spaces and M as in Example 4.9. If $\mathcal{S}$ $:=$ the class of all Kasahara spaces, then $(\mathcal{U}, \mathcal{S}, M)$ is, by the contraction principle, a f.p.s. It is an open problem if $\mathcal{S}_{max} = \mathcal{S}$ or not (see [14], [16], [18]).

5. Conclusions

In this paper we have studied the concept of set-mapping pair bijections in connection with the isomorphisms structure, and we have investigated their impact on the fixed point theory in a set-mapping pair in terms of Kasahara spaces. Some open problems were given (see Problem 2.1, Problem 2.2 and Example 4.10). Beside these problems, it would be interesting to conduct a similar study in the case of multivalued mappings. Regarding the fixed point structures for multivalued mappings on a set with some structure, see [14] and [16] and the references therein.

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