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Existence and uniqueness of solution of a tripled system of fractional Langevin differential equations with cyclic boundary conditions

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ABSTRACT. The present work examines the solvability of a tripled system of fractional Langevin differential equations with cyclic antiperiodic boundary conditions. The Krasnoselskii fixed point theorem, the Banach contraction mapping theorem, and specific properties of the Mittag-Leffler functions are employed to establish sufficient conditions for the existence and uniqueness of solutions. The feasibility of the primary findings is illustrated through the discussion of several numerical examples.

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