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An inertial method for solving the split equality fixed point problem with multiple output sets

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ABSTRACT. In this paper, we introduce the split equality fixed point problem with multiple output sets in real Hilbert spaces and propose an iterative method for solving the problem. We then establish a strong convergence result under the assumption that the underlying mappings are uniformly continuous quasi-pseudocontractive. We give some specific cases of our main result and finally provide a numerical example to reveal the effectiveness of our method. Our result extends many of the results in the literature.

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